# Dynamical reduction of discrete systems based on the renormalization-group method

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The renormalization-group (RG) method is extended for a global asymptotic analysis of discrete systems. We show that the RG equation in discretized form leads to difference equations corresponding to the Stuart-Landau or Ginzburg-Landau equations. We propose a discretization scheme which leads to a faithful discretization of the reduced dynamics of the original differential equations. [S1063-651X(98)01504-9]

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It is a fundamental problem in physics, especially in statistical physics, since Boltzmann, to reduce many degrees of freedom of a dynamical system to fewer degrees of freedom preserving the essential nature of the system [1]. The reduced dynamics is often described by a few collective variables which represent slow and long-wavelength motions of the system. There are several methods for the dynamical reduction such as the multiple-scale methods (including the reductive perturbation method [2,3]), the average methods (including Whitham method [4]), the method of normal forms [5], and so on. We are usually interested in the asymptotic behavior of the system after a long time. Thus the problem is to obtain a dynamical reduction in asymptotic and global domains.

Recently it was recognized and emphasized by Chen, Goldenfeld, and Oono [7] that the renormalization-group (RG) equation first developed in quantum field theory [6] is a powerful tool for global and asymptotic analysis: They applied the perturbative RG equation to ordinary and partial differential equations and showed that the RG equation nicely gives a reduction of the dynamics describing slow and long-wavelength motions of the system; the reduced dynamics is described by the so-called amplitude equations. Afterwards the reason for the powerfulness of the RG equations was accounted for in the context of the classical theory of envelopes [8]. More recently, it was shown that the RG method can also naturally lead to phase equations [9].

The purpose of this paper is twofold: One is to show that the RG method can be extended to *discrete* systems, and leads to dynamical reduction of *discrete* dynamical systems (maps). A discrete system is described by a *difference* equation. We notice that the extension is highly nontrivial because the applicability of the RG method for differential equations essentially relies on the local nature of differentiation [8]. Another purpose is to propose a discretization scheme for differential equations: Needless to say, different discretization schemes of nonlinear differential equations lead to different dynamical systems. We shall show that such a discretization-scheme dependence also exists for the reducibility of the dynamics, and gives a good discretization scheme based on the notion of the reducibility of the dynamics.

Now let us take the following discrete system as a typical example of nonlinear discrete systems,

$$x_{n+2}-2\cos\,\theta x_{n+1}+x_n=\epsilon f(x_n,x_{n+1}), \tag{1}$$

where $\theta$ is a constant and the function $f(x_n,x_{n+1})$ contains nonlinear terms. We remark that the equation can be converted to a vector equation, $\mathbf{X}_n=\mathbf{F}(\mathbf{X}_n)$, with $\mathbf{X}_n={}^t(x_n,y_n\equiv x_{n+1})$ and $\mathbf{F}(\mathbf{X}_n)={}^t(y_n,2\cos\theta y_n-x_n+\epsilon f(x_n,y_n))$. We have taken an example of a second-order equation here, but the following discussion is applicable for higher-order equations. We notice that the unperturbed equation ($\epsilon=0$) has a neutrally stable solution $\exp(\pm i\theta)$ [10]. Assuming that $\epsilon$ is small, we shall apply perturbation theory. To make the discussion definite, let us take a concrete form for $f(x_n,x_{n+1})$ as $f(x_n,x_{n+1})=a_1x_n+a_2x_n^2+a_3x_n^3$. If we set $a_2=0$, the resultant equation may be considered as a discrete version of the damped Duffin equation.

Expanding $x_n$ as $x_n=x_n^{(0)}+\epsilon x_n^{(1)}+\epsilon^2x_n^{(2)}+\cdots$, let us try to find a solution which is valid around $n=n_0$ where $n_0$ is arbitrary: $x_n^{(i)}$ $(i=0,1,\ldots)$ satisfies $\hat{L}x_n^{(0)}=0$, $\hat{L}x_n^{(1)}=a_1x_n^{(0)}+a_2x_n^{(0)2}+a_3x_n^{(0)3}$, and so on. Here $\hat{L}=E^2-2(\cos\theta)E+1$, with $E$ is the forwarding operator, i.e., $Ex_n=x_{n+1}$. The lowest order solution may be written as

$$x_n^{(0)}=A(n_0)e^{in\theta}+\text{c.c.} \tag{2}$$

Here we have made it explicit that $A$ may be a function of $n_0$; its functional dependence is as yet unknown, and will be determined by the RG equation, the determination of which constitutes the central part of the RG method. The first order correction $x_n^{(1)}$ is given by

$$x_n^{(1)}=-\frac{i}{2\sin\,\theta}(n-n_0)(a_1A+3a_3|A|^2A)e^{i(n-1)\theta}+\text{c.c.}, \tag{3}$$

where we have omitted nonsingular terms which are proportional to $\exp(ikn\theta)$ only with $k=0$, 2, and 3. Notice that there appears a secular term proportional to $n-n_0$. We remark that the solution of the first-order equation is not unique; one could add any term proportional to $x_n^{(0)}$. We have chosen the above form so that only new independent terms appear, and the secular term vanishes at $n=n_0$, which assures the lowest order approximation is as good as possible. Up to $O(\epsilon^2)$, we have an approximate solution $x_n=x_n^{(0)}+\epsilon x_n^{(1)}\equiv x_n((A(n_0);n_0)$, which is only locally valid around $n=\forall n_0$; the validity as an approximate solution is lost due to the secular term, as $|n-n_0|$ becomes large. This means that the naive perturbation expansion breaks down, which is a well known fact.

One may say, however, that we have a family of discrete curves $x_n(A(n_0),n_0)$ with $n_0$ being a parameter characterizing the curves [8]. Each curve will become a good approximation for the exact solution around $n=n_0$ if $A(n_0)$ is suitably chosen. Therefore the ''envelope'' of the family of the curves may give a good approximation of the exact solution in a global domain [12]. The ''envelope'' of $x_n((A(n_0);n_0)$ is constructed as follows: We first impose that

$$\Delta_{n_0} x_n(A(n_0);n_0)=0, \tag{4}$$

where $\Delta_{n_0}$ denotes the difference with respect to $n_0$. This is the basic equation of our method and we call it the RG/E equation. This is an equation to give $n_0$ and $A(n_0)$ as functions of $n_0(n)$. The ''envelope'' $x_n^E$ is given by inserting this solution into $x_n(A(n_0);n_0)$; $x_n^E=x_n[A(n_0(n));n_0(n)]$. However, since we are constructing the ''envelope'' which contacts with the local solutions at $n=n_0$, so that the ''envelope'' give a good approximation, $n_0$ should be $n$, i.e., $n_0=n$: Notice that this choice nicely eliminates the secular term from $x_n^E$. Conversely speaking, $A(n)$ can be determined so that Eq. (4) gives $n_0=n$; this possibility is related with the ''renormalizability'' of the equation [7]. Thus we have

$$\Delta A(n)=\epsilon\frac{\exp(-i\theta)}{2i\sin\theta}(a_1A(n)+3a_3|A(n)|^2A(n)), \tag{5}$$

where $\Delta$ denotes the difference operator with respect to $n$ [13]. This is the amplitude equation in a discrete form corresponding to Stuart-Landau equation [14] for continuum systems. $x_n^E$ is thus given by

$$x_n^E=(A(n)e^{in\theta}+\text{c.c.})+2a_2\epsilon|A(n)|^2+(\text{higher harmonics}). \tag{6}$$

A significant point is that this function gives an approximate but uniformly valid solution in a global domain. Let us show this in a general setting. Let $\mathbf{X}_n={}^t(X_{1n},X_{2n},\ldots,X_{dn})$ and $\mathbf{F}(\mathbf{X}_n,n)={}^t(F_1(\mathbf{X}_n,n),\ F_2(\mathbf{X}_n,n),\ldots,F_d(\mathbf{X}_n,n))$; we assume that $\mathbf{F}(\mathbf{X}_n,n)$ is analytic with respect to $\mathbf{X}_n$. We consider the difference equation

$$\Delta\mathbf{X}_n=\mathbf{F}(\mathbf{X}_n,n). \tag{7}$$

We suppose that $\tilde{\mathbf{X}}_n(\mathbf{W}(n_0),n_0)$ is an approximate solution to the equation up to $O(\epsilon^p)$, where the $d$-dimensional vector $\mathbf{W}(n_0)$ denotes the initial values assigned at the initial time $n=n_0$. Here notice that $n_0$ is arbitrary. The envelope function is given by $\mathbf{X}_n^E\equiv\tilde{\mathbf{X}}_n(\mathbf{W}(n),n)$, where $\mathbf{W}(n)$ is the solution to the RG/E equation, $\Delta_{n_0}\tilde{\mathbf{X}}_n(\mathbf{W}(n_0),n_0)=O(\epsilon^p)$. Then one can show that $X_n^E$ satisfies the original equation uniformly up to $O(\epsilon^p)$ as follows: $\Delta\mathbf{X}_n^E=\Delta\tilde{\mathbf{X}}_n(\mathbf{W}(n_0),n_0)|_{n_0=n+1}+\Delta_{n_0}\tilde{\mathbf{X}}_n(\mathbf{W}(n_0),\ n_0)|_{n_0=n}$ $=\mathbf{F}(\tilde{\mathbf{X}}_n(\mathbf{W}(n_0,n_0),n)|_{n_0=n+1}+O(\epsilon^p)=\mathbf{F}(\tilde{\mathbf{X}}_n(\mathbf{W}(n),n),n)$ $-\partial\mathbf{F}/\partial\tilde{\mathbf{X}}n\cdot\Delta_{n_0}\tilde{\mathbf{X}}_n(\mathbf{W}(n_0),n_0)|_{n_0=n}+O(\epsilon^{m\geqslant p})=\mathbf{F}(\mathbf{X}_n^E,n)$ $+O(\epsilon^p)$. Here we have utilized the RG/E equation in the second and last equality, and the analyticity of $\mathbf{F}$ in the third equality. This concludes the proof.

We notice that the amplitude equation (5) is a first-order equation and a dynamical reduction is achieved in comparison with the original equation. In fact, in the polar representation $A(n)=R_n\exp(i\varphi_n)$, $R_n$ and $\varphi_n$ satisfy the equations

$$R_{n+1}=R_n-(\epsilon/2)(a_1R_n+3a_3R_n^3) \tag{8}$$

and $\varphi_{n+1}=\varphi_n-(\epsilon/2)\cot\theta(a_1+3a_3R_n^2)$, respectively. Notice that $R_n$ is determined by the first order equation independently of $\varphi_n$, which in turn is given in terms of $R_n$. Thus one may say that the second order dynamical system Eq. (1) is reduced to a first order one. As seen from the derivation, this reduced equation has a universal nature, as has the Stuart-Landau equation.

The first order equation for $R_n$ has simple qualitative properties depending on the signs and values of $a_1$ and $a_3$. For example, when $a_1<0$ but $a_3>0$, the equation is converted to $f_{n+1}=f_n+af_n(1-f_n^2)$ with $f_n=\sqrt{3a_3/|a_1|}R_n$ and $a\equiv\epsilon|a_1|/2$. For $0<a<1$, the equation has a fixed point $f^*=1$, while, for $1<a\sim1.246$, the map shows a two-period behavior, and after that the map rapidly shows that multiple-period behavior then eventually becomes chaotic.

As another example of ordinary difference equation, let us take the one which is derived as a discretization of the Rayleigh equation: $\ddot{x}+x=\epsilon\dot{x}(1-1/3\dot{x}^2)$. We remark that the equation admit a limit cycle with the radius of 2 [15]. We take the following discretizations; $\ddot{x}\to(x_{n+1}-2x_n+x_{n-1})/\Delta t^2$ and $\dot{x}\to(x_n-x_{n-1})/\Delta t$. That is, the central difference for the second derivative and the backward difference for the first derivative. Thus we have

$$x_{n+1}-2\cos\theta x_n+x_{n-1}=\bar{\epsilon}(x_n-x_{n-1})\left(1-\frac{1}{3}\frac{(x_n-x_{n-1})^2}{\Delta t^2}\right), \tag{9}$$

where $\cos\theta=1-\Delta t^2/2$ and $\bar{\epsilon}=\epsilon\Delta t$. We remark that this difference equation has a neutrally stable solution like the unperturbed one. This is not the case for other discretization schemes such as $\ddot{x}\to(x_{n+2}-2x_{n+1}+x_n)/\Delta t^2$ and $\dot{x}\to(x_{n+1}-x_n)/\Delta t$. We put $1/\Delta t=\omega$.

In the first order approximation with respect to $\bar{\epsilon}$, we have

$$\begin{aligned}x_n&\equiv x_n(A(n_0);n_0)\\&=\left[A(n_0)+\bar{\epsilon}\frac{\exp(-i\theta)}{2i\sin\theta}(1-\omega^2|B(n_0)|^2)\times B(n_0)(n-n_0)\right]e^{in\theta}-\frac{\bar{\epsilon}}{3}\omega^2\\&\quad\times\frac{B(n_0)^3\exp(3in\theta)}{(\exp(3i\theta)-\exp(i\theta))(\exp(3i\theta)-\exp(-i\theta))}+\text{c.c.},\end{aligned} \tag{10}$$

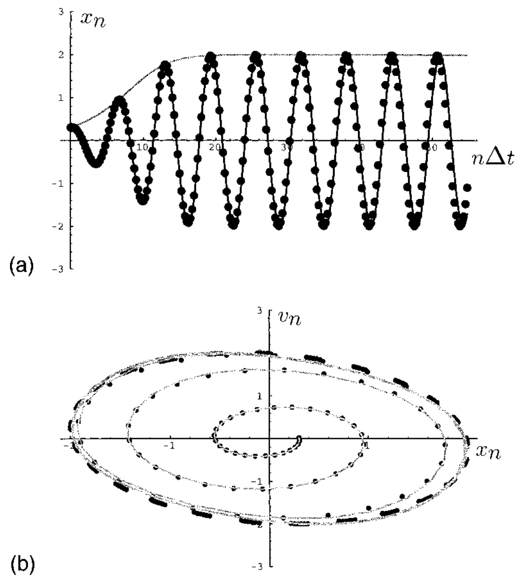


FIG. 1. (a) The dots show $x_n^E$ [Eq. (12)] vs $n\Delta t$, while the thin line shows the envelope $2R_n$ for $\epsilon=0.4$ and $\Delta t=0.25$. The bold line shows the exact solution of Eq. (9). (b) The behaviors in ''phase space.'' The vertical axis denotes the ''velocity'' $v_n \equiv (x_{n+1}-x_n)/\Delta t$, while the horizontal axis denotes $x_n$. The dots show our approximate solution, and the solid line shows the exact one.

where $B(n_0)=(\exp(i\theta)-1)A(n_0)$. Now the RG/E equation $\Delta_{n_0} x_n(A(n_0);n_0)|_{n_0=n}=0$ gives the amplitude equation $\Delta A(n)=\overline{\epsilon}\exp(-i\theta)/2i\sin\theta(1-\omega^2|B(n)|^2)B(n)$; accordingly,

$$A(n+1)=A(n)+zA(n)(1-|A(n)|^2), \tag{11}$$

with $z\equiv\overline{\epsilon}\exp(-i\theta)/\cos\theta/2$. If we take the polar representation $A(n)=R_n\exp(i\varphi_n)$, we have $R_{n+1}=R_n+\epsilon a' R_n(1-R_n^2)$, and $\varphi_{n+1}=\varphi_n+\overline{\epsilon}(\sin\theta/2)(1-R_n^2)$ with $a'=\Delta t\cos\theta/(2\cos\theta/2)$. We note that $a'$ as a function of $\Delta t$ is a parabolalike shape, and takes a maximum about 0.6 at $\Delta t\simeq 0.9$; it vanishes at $\Delta t=0$ and $\Delta t\simeq 1.4$. Thus we see that $R_n$ goes up monotonically to a fixed point 1.

With this $R_n$ and $\varphi_n$, the envelope $x_n^E$ is given as

$$x_n^E \equiv x_n(A(n);n)=2R_n\cos(n\theta+\varphi_n) + \frac{\overline{\epsilon}}{12}\frac{\tan\theta}{\cos^3\theta/2}R_n^3\sin\{3(n\theta+\varphi_n)-\tfrac{3}{2}\theta\}, \tag{12}$$

which shows that the radius of the limit cycle is 2, irrespective of the choice of $\Delta t$ in accordance with the original Rayleigh equation. We remark that it is not the case for other discretization schemes as given below Eq. (9).

Figure 1(a) shows $x_n^E$ given by Eq. (12) and the envelope $2R_n$ together with the exact solution of Eq. (9) with $\epsilon=0.4$ and $\Delta t=0.25$. One can see that the agreement is excellent in the global domain; notice that the result is obtained in the first order approximation. One can also see that the amplitude $2R_n$ successfully describes the slow motion of the system. The characteristic features of the system as a dynamical system may be more clearly seen in Fig. 1(b), where the behaviors in the ''phase space'' are shown. The agreement is again excellent [16].

Finally, we consider partial difference equations. As an example, we take the difference equation which is given by a discretization of the one-dimensional Swift-Hohenberg equation [17]: $\partial_t\phi(x,t)=\epsilon[\phi(x,t)-\phi(x,t)^3]-(1+\partial_x^2)^2\phi(x,t)$. With the discretization $\partial_t\phi\rightarrow[\phi(n,m+1)-\phi(n,m)]/\Delta t\equiv\Delta_m\phi(n,m)]/\Delta t, \partial_x^2\phi\rightarrow[\phi(n+1,m)-2\phi(n,m)+\phi(n-1,m)]/\Delta x^2\equiv\Delta_n^2\phi(n,m)/\Delta x^2$, we have the following partial difference equation:

$$\hat{\mathcal{L}}\phi(n,m)=\epsilon[\phi(n,m)-\phi(n,m)^3], \tag{13}$$

where $\hat{\mathcal{L}}\equiv\Delta_m+r(\Delta x^2+\Delta_n^2)^2$, with $r\equiv\Delta t/\Delta x^4$. We shall show that the difference equation admit a dynamical reduction giving an amplitude equation which is the analogue of the time-dependent Ginzburg-Landau equation in the continuum theory. The reason why the dynamical reduction is possible in the RG method is that the difference equation is constructed so that the unperturbed equation has a neutrally stable solution.

Making the Taylor expansion $\phi=\phi_0+\epsilon\phi_1+\epsilon^2\phi_2+\cdots$, we have equations in the successive order $\hat{\mathcal{L}}\phi_0=0$, $\hat{\mathcal{L}}\phi_1=\phi_0-\phi_0^3$, and so on. We consider an asymptotic solution at $m\rightarrow\infty$ and take the following neutrally stable solution as the zeroth order one

$$\phi_0(n,m)=A(n_0,m_0)e^{in\theta_x}+\text{c.c.}, \tag{14}$$

with $\theta_x=2\sin^{-1}\Delta x/2$. Here we have made it explicit that the amplitude $A$ may depend in an as yet unknown way on the initial time $m_0$ and point $n_0$. They will be determined by the RG/E equation.

Then the first-order equation now reads

$$\hat{\mathcal{L}}\phi_1=\{(A-3|A|^2A)e^{in\theta_x}-A^3e^{3in\theta_x}\}+\text{c.c.} \tag{15}$$

A straightforward but somewhat tedious manipulation gives

$$\phi_1(n,m)=\left\{\left[\mu_1(m-m_0)-\frac{\mu_2 r^{-1}}{8\sin^2\theta_x}\times\{n^{(2)}-n_0^{(2)}+ie^{-i\theta_x}(n-n_0)\}\right]\times(A-3|A|^2A)e^{in\theta_x}-\frac{A^3e^{3i(n+2)\theta_x}}{r(e^{3i\theta_x}-e^{i\theta_x})^2(e^{3i\theta_x}-e^{-i\theta_x})^2}\right\}+\text{c.c.}, \tag{16}$$

where $n^{(2)}=n(n-1)$ and $\mu_1+\mu_2=1$. Thus we have an approximate solution $\phi(n,m)=\phi_0(n,m)+\epsilon\phi_1(n,m)$, which is valid only for $(n,m)$ around $n\sim n_0$ and $m\sim m_0$.

Now the RG/E equations $\Delta_{m_0}\phi|_{m_0=m}=0$ and $\Delta_{n_0}\phi|_{n_0=n}=0$ give $\Delta_m A(n,m)=\epsilon\mu_1(1-3|A|^2)A$, and $\Delta_n^2 A(n,m)=-\epsilon\mu_2/4r\sin^2\theta_x(1-3|A|^2)A$, respectively. Here we have utilized the fact $\Delta_m A=O(\epsilon)$ and $\Delta_n A=O(\epsilon)$, and neglected terms of $O(\epsilon^2)$. Thus, noting that $\mu_1+\mu_2=1$, we reach the amplitude equation for the difference equation

$$\Delta_m A(n,m) = 4r \sin^2\theta_x \Delta_n^2 A(n,m) + \epsilon(1-3|A(n,m)|^2)A(n,m). \quad (17)$$

This is precisely the discretized form of the time-dependent Ginzburg-Landau equation: $\partial_t A(x,t) = 4\partial_x^2 \phi(x,t) + \epsilon(1-3|A(x,t)|^2)A(x,t)$.

In summary, we have shown that the renormalization group method can be nicely extended to discrete systems, and that the method is useful as a tool for global asymptotic analysis and gives dynamical reduction of discrete systems. We have emphasized that the method is applicable for systems which have neutrally stable solutions. It is to be remarked that this is also the case for equations which have unperturbed solutions on invariant stable and unstable manifolds. We have also given a notion of the discretization scheme that faithfully preserves the nature of the reduced dynamics irrespective of the magnitude of $\Delta t$. Finally, we notice that it is not trivial which discretization scheme preserves the integrability of differential equations which admit soliton solutions [18]. It will be interesting to examine if our discretization scheme based on the RG method can have a relevance to soliton theories.

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